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I

DYNAMICAL ASPECTS

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In opening this symposium I can undertake to do no more than explain, in a most elementary way, the dynamical principles upon which artificial flight depends. It is difficult to do this without the use of differential equations, which would be out of place in a popular discussion, so that my treatment must confine itself to the merest outline. We must distinguish at the outset between *aëronautics* properly so-called, in which we have to do with airships, that is apparatus possessing natural sustentation through the buoyancy of the air displaced, which is at least as heavy as the airship, and *aviation,* which is the operation of apparatus that has no natural sustentation or buoyancy, being heavier than the displaced air, and, like a bird, possessing sustentation only when in motion. Unfortunately we have no generic term for the latter apparatus, corresponding to the recently coined French word "*avion,*" and we are obliged to make use of the word aëroplane, although the term plane is not always accurate. While the principle of Archimedes, namely that a body is buoyed up with a force equal to the weight of the displaced fluid, this force acting at a point coincident with the center of mass of the fluid displaced, is sufficient for the study of the equilibrium of the airship, totally different principles are involved in connection with the aëroplane.

The first principle that we shall use is that of relative motion of the aëroplane and the air. It will be admitted that the forces involved are the same whether, as in the case of the kite, the object is at rest and the air in motion, or as in the case of the aëro-

plane the air is at rest and the object in motion in the direction opposite to that of the preceding case. We also notice that in both cases three forces are involved, first, the weight of the object, second, the action of the wind on the plane, and third, the pull of the kite-string or the thrust of the propeller. I may also say that it makes no difference whether the propeller pushes from behind, as in the first aëroplanes, or pulls from in front, as is now usually the case.

Since the time of Newton it has been known that the force of the wind on the plane is proportional to the square of the relative velocity, since it is proportional to the momentum destroyed in a given time, and this is proportional, for a given mass, to its velocity, while the mass arriving is again proportional to the velocity, so that the square is accounted for. Finally the influence of the angle made by the wind with the surface of the plane, the so-called angle of attack, must be known. We may assume that wind blowing tangent to a surface will slide along without exerting any force on it, although the action of the wind in supporting a flag shows that this is not so. But the wing of an aëroplane is made so smooth that for practical purposes we may neglect the tangential drag, and as-sume that the force is at right angles or normal to the plane. Ac-cording to Newton, who treated the air like a stream of particles impinging on the plane, the force would have been proportional to

FIG. I.
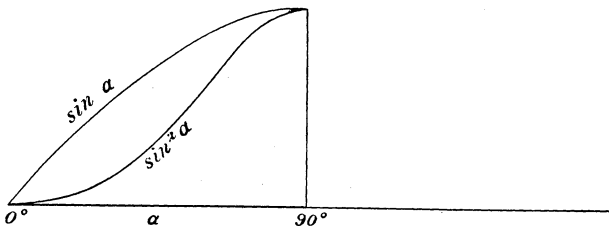
the square of the sine of the angle of attack, but we now know through the many series of experiments that have been made by Langley and others, that this law is not correct, and that it is much more nearly proportional to the first power of the sine. The dif-ference is made apparent in Fig. I, in which the vertical height of a point denotes the force, the horizontal distance the angle of at-tack of the plane, for both laws. We see that for small angles the

sine law gives a much more rapid increase of force than the sine-square, which is a very important point in practice.

Beside the force at right angles to the plane the current tends to turn the plane about a certain axis, as we see if we drop a card with its long dimension horizontal. In falling it turns over and over even if started with its surface horizontal. This turning effect may be explained if we draw the stream-lines, which show at each point the direction of flow of the air. It is a proposition due to Bernoulli, that where the flow is fast the pressure is small, and where it is slow the pressure is great. In Fig. 2 where the
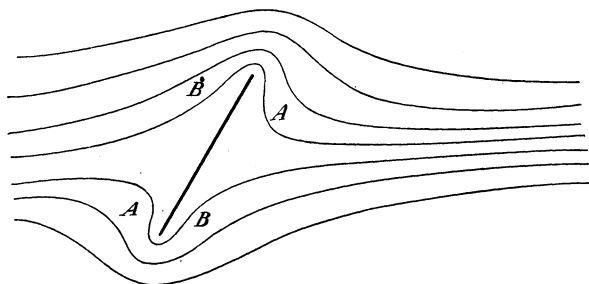

FIG. 2.

stream lines are far apart the flow is slower than where they are near together, just as a river flows most slowly where it is widest, so that the pressure is large in such points as A, A, and small at B, B, where the flow is rapid. Thus there is a tendency to turn the body in the direction of the arrow. We may express this turning property by saying that the effect of the air current on the plane is represented by a single force R applied at a point P called the center of pressure, not at the center of the plane, the position of P varying according to the angle of attack.

Much mathematical skill has been expended to determine the law of variation of the force with the angle, and the position of the center of pressure. Curiously enough if the air acts like a perfect fluid, and does not form vortices, it can be shown that there would be no force on an obstacle, but merely a turning moment. But if there are surfaces where the motion is discontinuous, on crossing which we pass from fluid that is moving to fluid that is at rest or moving less rapidly, the forces can be accounted for. Kirchhoff many years ago treated such motions, and Sir George Green-

hill has followed him in working out a great number of cases with great skill. In Fig. 3 we see the flow past a cambered wing, with stream-lines continuous in Fig. *a,* causing no pressure, and in Fig. *b* with the stream splitting along the dotted line, part going up and

FIG. 3*a.*

part down, with discontinuity along the lines *AB, CD,* between which the fluid is comparatively at rest. From this assumption of the flow it is possible to calculate the thrust and the turning. But even this assumption about the flow is not true in practice, but the air forms vortices, which cause a calculation to be still more
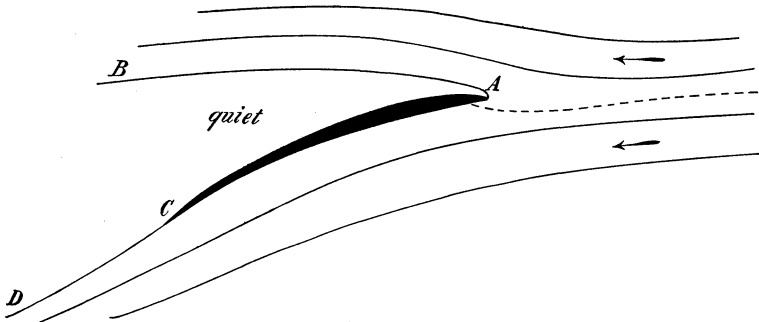

FIG. 3*b.*

difficult. Accordingly it becomes necessary to determine the laws of pressure by actual experiments on small scale models in wind tunnels, such as those of M. Eiffel in Paris, Professor Prandtl in Göttingen, Professor Joukowsky in Moscow, or that at the Massachusetts Institute of Technology used by Mr. Hunsaker in his experiments. In all these cases a steady stream of air is caused to flow through the tunnel by means of a blower, and the model is hung in the wind upon balances which enable the forces, their points of application and direction to be carefully measured for all angles of attack. We may expect in the next few years to see many such wind-tunnels constructed in this country, and large increases made in our experimental knowledge.

Suppose we now know the law of the force exerted by the air current on the plane, and the position of the center of pressure. We have now to apply an elementary principle of equilibrium of rigid bodies. If a body is submitted to the action of three forces the lines of action of these forces must pass through a common point. Thus if we consider a single plane supporting a machine, with the resultant pressure R, Fig. 4, with weight W concentrated at the center of gravity of the whole machine G, the thrust of the propeller D, which is nearly horizontal, must pass through the intersection of R and W. The second principle is that if we draw lines representing by their length and direction the three forces in
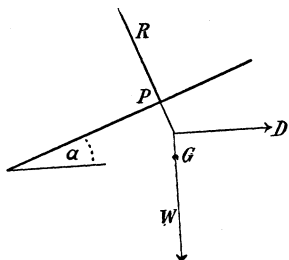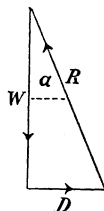

FIG. 4.   FIG. 5.

question, these lines must form a closed triangle, Fig. 5. Thus knowing the weight W, we may find D, the thrust required from the motor, as well as R, the force required, and a the angle of attack.

I shall illustrate the preceding principles by a very simple experiment, which I think well shows all the leading ideas involved in the dynamics of the aëroplane. I have here a heavy card fastened by a hook at the middle of one side to this rubber cord. I now need a very brave assistant, whom I request to hold the end of the rubber cord at the height of his shoulder. I strongly stretch the cord, holding the card in my hand, both card and string being horizontal. We are thus in a position to perform the Wilhelm Tell experiment, with the apparent probability that, since there are absolutely no upward forces present, the card will cut Walther's head off. On releasing the card you see that no such thing happens, but the card soars several feet above my assistant's head, although the cord is actually pulling down all the time. The reason is that

on release the card immediately tips downward behind, and as it goes ahead with great velocity receives more than enough upthrust for its own sustentation, and is actually able to rise, although pulled down by the string.

I come now to perhaps the most important dynamical aspect of aviation, that is the question of stability of flight. Stability of equilibrium is a familiar notion, and exists when a system, if displaced, tends to return to its former position, generally performing small oscillations about it which die away, leaving it in its equilibrium position. Thus a pea at the bottom of a bowl is in stable equilibrium, but on top of a sphere, though in equilibrium, is unstable,
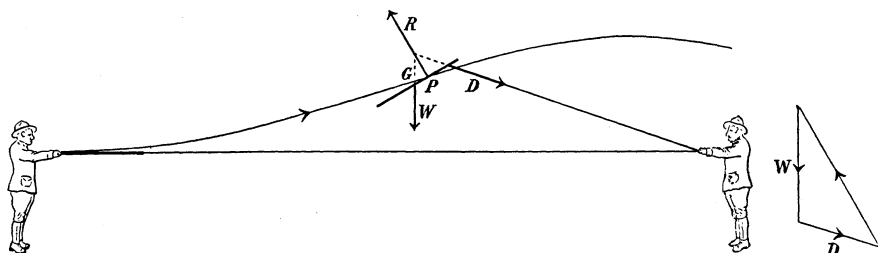

FIG. 6.   FIG. 6a.

because if slightly displaced it will not return, but will roll off. Stability of motion may be similarly defined. If an aëroplane is in flight, and is slightly displaced in position or direction, will it tend to resume its position or will it tend to leave it more and more? Consider what happens when it tips forward and downward. If the center of pressure moves forward when the angle of attack is less it will tend to turn the plane backward, so as to resume its former position. So far then the motion is stable. As it tips forward the angle of attack becomes smaller, the sustaining force becomes less, and the aëroplane sinks, but when tipped back again it rises once more. Thus the path oscillates about a horizontal line. But a rigid body has six ways of moving: forward and back, sidewise right and left, and vertically up and down, making three, together with three ways of turning, rolling about an axis fore and aft, pitching about a transverse axis, and yawing, or turning about the vertical. If any of these six motions are disturbed, how will the motion be affected? It is easily shown that a change in any of

these six motions affects all the others, as already shown for pitching and rising. In treating this problem we use differential equations invented by Euler for problems in which we have to do with rotating axes of coördinates, and we are thus able to find the mutual connection of the different sorts of motion. Now if the disturbances are small, we are able to use the method introduced by Lagrange in his famous " Mécanique Analytique " for the treatment of small oscillations, which leads to the introduction of an algebraic equation of degree twice as great as the number of degrees of freedom of the system, in our case six, so that the equation would be of the twelfth degree. On account of symmetry, however, our equation reduces to degree eight, and falls apart into two equations of degree four. It is useless to undertake the general solution of these, but when we have the constants of a given apparatus, as determined by experiment, it is possible to solve the equations arithmetically with any desired degree of approximation. This is what has been done by various investigators, like Bryan and Bairstow in England, and Professor E. B. Wilson here. In fact when this work has proceeded to a certain extent, it is no longer necessary to have recourse to learned mathematicians, but it may be farmed out
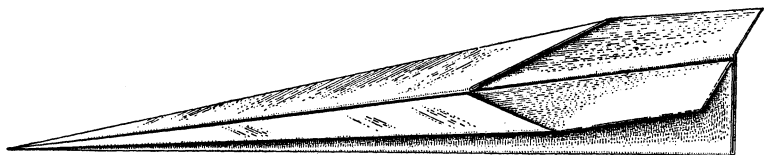

FIG. 7.

to computers, so as to be greatly expedited, and thus the design of machines may be greatly improved. I may say that machines generally gain more stability with greater speed, and that too great stability is not desirable, as it would lead to difficulty in steering or rising. At any rate the theory has now arrived at such a stage that we may hope to avoid such accidents as formerly occurred in great numbers owing to improper design.

I will conclude with a simple experiment showing the intrinsic stability possessed by a very simple aëroplane such as I learned to make when a schoolboy, which I am able to fold from a piece of paper before your eyes and to throw with a good deal of accuracy.